Peer-to-Peer Energy Trading Enabled Optimal Decentralized Operation of Smart Distribution Grids

Lahanda Purage Mohasha Isuru Sampath, Member, IEEE, Amrit Paudel, Member, IEEE,
Hung D. Nguyen, Member, IEEE, Eddy Y. S. Foo, Member, IEEE,
and Hoay Beng Gooi, Life Senior Member, IEEE

Abstract—Currently, the distribution systems are moving towards decentralized operation due to the high penetration of distributed energy resources (DERs). Peer-to-peer (P2P) energy trading has been an emerging concept that promotes autonomous DER participation in energy markets while preserving their privacy concerns. In this work, a novel P2P energy trading enabled decentralized market framework is proposed for the optimal operation of distribution grids. Nodal agents and P2P agents are established as market participants, and market equilibrium is iteratively achieved via alternating direction method of multipliers based algorithms. The proposed market framework guarantees grid constraint satisfaction, market equilibrium, and global optimality for all market participants without violating their privacy concerns. The agent coordination and local optimization are designed such that fairness of the market clearing mechanism, prosumer autonomy, and prosumer anonymity is preserved without compromising the market efficiency. Further, costs/rewards of ancillary services associated with the P2P energy transactions are considered as trade-offs within the market mechanism, and those are accurately allocated to the respective trading pairs. The case studies illustrate the effectiveness and scalability of the proposed market framework.

Index Terms—Ancillary service provisions, decentralized operation, optimal power flow, peer-to-peer energy trading.

I. INTRODUCTION

Decentralized market concepts such as multi-regional grid operation and peer-to-peer (P2P) energy trading are emerging in distribution grids due to the increased penetration of distributed energy resources (DERs), and the smart features like advancements in energy storage systems (ESSs) and communication technologies [1]–[4]. Further, the integration of the large number of flexible loads supported by the demand-side management schemes promotes decentralized operation. This results in a proliferation of prosumers in modern distribution grids who own several DERs, ESSs, flexible loads, etc., and have the capability to produce or consume energy at a particular market interval depending on their preference [5]–[7]. Traditionally, the energy management of the distribution grid is a centralized operation facilitated by the distribution system operator (DSO) by executing an optimal power flow (OPF). The prosumers in distribution grids sell (or purchase) energy at the feed-in (or retail) tariff specified by the central market operator/DSO [8]–[12]. For energy management and decentralized coordination between individual prosumers and the DSO, distribution locational marginal price (DLMP)-based optimization schemes are proposed in [13], [14]; a network-constrained Stackelberg game is proposed in [15]; and an inner-outer iteration approach is proposed for prosumers as end-user aggregators in [5].

A. P2P Energy Trading in Distribution Grids

P2P energy trading enables prosumers to establish contractual agreements with other prosumers by directly negotiating for price and energy transaction amounts [7], [16]–[24]. Similarly, market frameworks for transactive energy trading in addition to the imports/exports from the utility have been proposed in [8]–[11] considering multi-microgrid systems. Typically, the prosumers demand trading in an independent, proactive, anonymous, and privacy-preserved manner without relying on a central operator. These privacy requirements may include cost/welfare functions, capability limits, and location information that prosumers would not be willing to share with other prosumers. Therefore, centralized optimization frameworks are not applicable for P2P energy trading [18]–[28]. For instance, game theoretic frameworks are proposed in [18], [19] to optimize the welfare of prosumers. An efficient pricing strategy is proposed in [20]. A peer-matching and negotiations mechanism is proposed in [21]. The trade weights are introduced in [22]. A scalable market design and bilateral contract networks are proposed in [23] introducing the forward and real-time markets. Community-based ESSs and Q-learning-based optimal pricing policy is proposed in [24]. A two-stage stochastic game model is developed in [25] while considering the conditional value-at-risk criterion for a set of individual multi-energy microgrids. A two-stage energy sharing strategy with a real-time optimization model is presented in [26] for a building cluster. A cooperative Stackelberg game is proposed in [27] that supports peak energy demand curtailment by forming a stable coalition of prosumers for local energy trading. A retail energy broker is proposed in [28] to facilitate indirect customer-to-customer energy trading using reinforcement learning techniques.

Typically, the prosumers (physically) locate at several buses/nodes in the distribution grid. Hence, the P2P energy...
transactions cause changes in power flows over the distribution grid and might induce grid constraint violations [2], [3], [16], [17]. A two-stage optimization model and a blockchain-based architecture are proposed for P2P energy trading with grid interactions in [6]. Therein, the incentive design routine for prosumers (or crowdsources) is formulated considering the distribution grid parameters and constraints. A cooperative energy market model is developed for P2P energy trading in distribution grids considering the social welfare maximization and energy trading based on the Generalized Nash Bargaining in [29]. However, in these energy market frameworks, the prosumer optimization models are formulated incorporating the topology parameters and the technical constraints of the distribution grid, which can be impractical as the DSO is not willing to share that information for security reasons. Therefore, implementing distributed P2P energy markets which satisfy the distribution grid constraints and achieve global optimality requires certain coordination and it is a challenging task [7], [17].

To address this concern, Ref. [7] presents a grid utilization fee calculation based on an electric distance method derived using the decoupled power flow approximation. Ref. [12] derives first-order sensitivities for voltage, power transfer, and losses via first-order approximations of AC power flow constraints. Ref. [30] presents a scalable P2P energy trading framework considering linear dist-flow-based voltage and line congestion management, and a mutual reputation index for efficient peer matching. Ref. [31] presents a graph-based loss allocation model with multi-layered radial graphs to capture the unbalance nature of power flows. Ref. [32] proposes a fair P2P energy trading framework to alleviate the cross-subsidization issues in loss allocations considering the first-order loss sensitivities. Ref. [33] introduces the energy collectives as a community-based electricity market structure. Ref. [34] develops a loss allocation policies for joint transmission and distribution P2P energy markets based on power transfer distribution factors. Ref. [35] presents a multi-round double auction-based P2P energy market framework with first-order sensitivities to check the violations of grid constraints. However, the market-clearing, which includes peer-matching, power balance, and the allocation of ancillary service costs is centrally managed while the DSO serving as the auction manager, compromises the security and fairness of the market clearing mechanism to a greater extent. In addition, the establishment of a third-party entity, such as market operator in [34] and DSO in [35], to determine the trading price/market clearing price disregard the autonomy and anonymity of prosumer operations. Ref. [36] suggests a method for allocating grid utilization fees for P2P participants based on the Thevenin electrical distance. Most importantly, the above-mentioned methods do not accurately represent the power flow equations. Hence, the DSO may not be sufficiently incentivized for provisioning ancillary services to satisfy the power losses and to avoid voltage violations and congestion issues. Further, [10] proposes a coordinator-aided (where the coordinator is an intermediate arbitrator between microgrids/prosumers and the DSO) energy market framework only considering the costs of power losses (avoids the costs of voltage and congestion management) associated with P2P energy transactions.

In contrast, P2P energy market architectures with DLMP-based grid utilization fees are proposed in [16] to share the costs/rewards of ancillary services associated with P2P energy transactions. As the P2P energy market frameworks in both [16] and [35] do not consider the DLMP variations against the P2P transaction amounts, [17] introduces an iterative algorithm with interpretable grid utilization fees via decomposition of DLMPs. However, the convergence of the P2P energy market mechanism in [17] is not guaranteed, as the dual variable/DLMP updates are derived satisfying the nonconvex Karush–Kuhn–Tucker condition [17, Remark III.3]. Conversely, a single-stage P2P energy market framework is proposed in [37] with probabilistic DLMPs and the application to a multi-phase distribution system. The uncertainty/variation of DLMPs against P2P energy transaction amounts are addressed by using a conservative approximation for DLMPs considering their mean and standard deviation values via point estimates. In this work, the grid constraint feasibility depends upon the choice of the price-spread scale factor; and the mixed-integer linear program models used may not guarantee the global optimality and the convergence of the P2P energy market mechanism for large-scale applications. Apart from the lack of convergence guarantees, approximation errors, and the conservative nature, one of the major limitations in the implementation of P2P energy market frameworks introduced in [16], [17], [37] is that the cost of ancillary service allocation is achieved while sharing the DLMPs of each prosumer with other prosumers. This not only introduces an additional communication burden but also partially compromises the anonymity of the prosumer participation in the P2P energy market mechanism.

Lastly, some of the prosumers in the P2P energy market might operate at lagging power factor conditions, whereas DERs and ESSs are inverter-interfaced devices that can support reactive power requirements. However, producing reactive power from inverters reduces their efficiency and lifetime of operation [38]. Hence, suitable incentive mechanisms are required to promote their participation in reactive support, which can significantly improve the efficiency of the P2P energy market. Although [32], [34] consider the reactive power dispatch of prosumers, they do not illustrates how it benefits for enhancing the P2P energy sharing between prosumers.

B. Decentralized Operation of Distribution Grids

Compared to the centralized grid operation, decentralized grid operation enhances modularity, scalability, and privacy [39], [40]. However, efficient energy market frameworks must be designed to establish the coordination between individual market operators and optimize their energy management functions enabling the decentralized grid operation [1], [39]. Therein, alternating direction method of multipliers (ADMM)-based coordination-optimization algorithms are popular in distributed convex optimization due to their promising convergence characteristics [41]. However, the OPF problem which determines the optimal operation of distribution systems is nonconvex and hence NP-hard to solve. A branch-flow model
combined with a second-order cone relaxation is proposed to resolve nonconvexity with guaranteed exactness for radial grid topologies in [42] under certain conditions. Therewith, an ADMM-based decentralized-OPF solution approach is developed in [40] considering the second-order cone relaxation. Furthermore, a multi-region OPF framework is developed in [39] with trust-region-based linear grid constraint approximation for local optimization and ADMM-based coordination between regions. References [34], [35] develop P2P energy trading frameworks while considering the interactions of multiple market operators. However, they neglect the costs of voltage and congestion management, and specifically constitute centralized pricing strategies, which ignore the autonomy and anonymity of prosumer participation, restricting their practicality. To the best of the authors’ knowledge, fully decentralized optimal operation of distribution grids, which enables P2P energy trading while preserving the prosumer autonomy and anonymity in market participation, is still unaddressed in the literature.

C. Contributions

To this end, the main contributions of this work are summarized as follows.

1) A novel unified energy market framework is proposed for the P2P energy trading enabled optimal decentralized operation of distribution grids while respecting the network operational constraints. The proposed market framework defines the functionality and interactions of several independent market participants for a fair share of operating costs while maximizing their welfare that guarantees grid-feasible market equilibrium.

2) The market clearing mechanism of the proposed energy market framework does not require prosumer-specific information such as DLMPs, loss sensitivity factors, power transfer distribution factors, etc., to achieve market equilibrium. Hence, the autonomy and anonymity of prosumer participation are ensured, strengthening the fairness of P2P energy trading without compensating the market efficiency or computational accuracy.

3) We analytically prove that market equilibrium is also a trade-off between the costs/rewards of ancillary services (i.e., the combined cost/reward of satisfying losses, adhering to voltage limits, and the grid congestion management) associated with the P2P energy transactions, in addition to the marginal cost of producers and the marginal welfare of consumers.

4) In the proposed market framework, P2P participants are incentivized based on their reactive power injections. Hence, a unique formulation of the P2P energy trading problem and a pricing strategy (i.e., prosumer optimization models and agent interaction mechanism) are realized compared to existing approaches. Further, the impact of reactive power contribution on the P2P market equilibrium is also analyzed.

In this work, the decentralized market clearing mechanism is designed with ADMM-based distributed optimization-coordination algorithms. Local optimization problems are developed preserving the convexity of the overall optimization problem. The results illustrate the efficiency and scalability of the proposed decentralized market framework based on standard test systems.

The organization of the paper is as follows. Section II describes the decentralized market framework, including the agent establishments and optimization problem formulations. Section III describes the ADMM-based market clearing mechanism. Section IV analyses the equilibrium between prosumer trading prices at the market equilibrium. Section V describes the test systems and performance analysis. Finally, Section VI concludes the paper.

Notation—The set of real numbers is denoted by $\mathbb{R}$, and the set of nonnegative real numbers by $\mathbb{R}_+$. For a matrix $A$, its transpose is $A^\top$; its element in row $i$ and column $j$ is $A_{i,j}$; its row $i$ is $A_i$; and its column $j$ is $A_{:,j}$. For vector $a$, $\bar{a}$ and $a$ are its maximum and minimum limits respectively. $\mathbf{1}_N$ is an $N \times 1$ dimensional vector of ones.

II. DECENTRALIZED MARKET FRAMEWORK

In this section, the distribution system operation and the P2P energy market mechanism are decomposed, and the agents/market participants are established to perform market operations (i.e., local optimization and agent coordination) in a decentralized manner. The optimization problems to be executed by each market participant are formulated considering the individual objectives, capability limitations, and technical constraints. The shared/negotiated information, and accordingly, the agent coordination are also defined. Then, the market clearing is regarded as an equilibrium problem, where all market participants negotiate their energy transactions and trading prices while maximizing individual profits.

A. Market Agent Establishment and Coordination

Let $\mathcal{N}$ be the set of buses, and $\mathcal{G}$ be the set of sources including the point-of-common-coupling (i.e., bus 1) and the local generation sources that provision ancillary services, i.e., loss satisfaction, voltage regulation, and congestion management, to the distribution grid respectively. Exploiting the radial topology, let $\mathcal{A}(n)$ and $\mathcal{C}(n)$ be the ancestor bus and the set of child/ descendant buses of bus $n \in \mathcal{N}$ in the distribution grid respectively with $\mathcal{A}(1) = \emptyset$. In the proposed decentralized market framework, a nodal agent is established corresponding to each node/bus of the distribution grid to execute retail market operations. Their identity is verified by the DSO and they maintain contractual agreements with DSO to locally execute the retail market operations at corresponding nodes. As such, the energy management of the distribution system is facilitated by $|\mathcal{N}|$ nodal agents in a distributed manner. These nodal agents are independent in decision making and are individually responsible to maintain the power balance of the corresponding node while satisfying its voltage limits and the power flow limits of the lines physically connected to it. Therefore, each nodal agent possesses partial information of the parameters and constraints of the grid topology which is relevant to its operations. Further, each nodal agent coordinates with the neighboring nodal agents that manage its physically
connected nodes (i.e., its ancestor bus and the set of descendant buses), aiming to maximize its profit while satisfying the grid constraints.

**B. Decentralized Energy Management of Distribution Grids**

In this section, the proposed P2P energy trading enabled OPF problem for distribution systems is formulated as several subproblems, which are optimized by the aforementioned nodal agents in a distributed manner. Let \( p^D, q^D \in \mathbb{R}^{\vert N \vert} \) be the active and reactive power demand vectors. The function \( T^G : G \rightarrow N \) maps the connectivity of \( G \) to the buses of the distribution grid with \( G(n) = \{ g \in G : T^G(g) = n \} \subseteq G \). Let \( p^G_n, q^G_n \in \mathbb{R}^{\vert G(n) \vert} \) be the active and reactive power generation dispatch vectors at bus \( n \) respectively. Then, let \( V_n \) be the voltage magnitude at bus \( n \) and \( w_n = V_n^2 \). Let \( r_n, x_n, I_n, P_n, Q_n, \) and \( F_n \) be the resistance, reactance, (complex) current flow, active power flow, reactive power flow, and power transfer capacity of the line between bus \( A(n) \) and bus \( n \) respectively; and \( \ell_n = \vert I_n \vert^2 \).

Let \( S \) be the set of sellers/producers and \( B \) be the set of buyers/consumers in the P2P energy trading market. The functions \( T^S : S \rightarrow N \) and \( T^B : B \rightarrow N \) map the connectivity of P2P agents to the buses of the distribution grid. Then, let \( S(n) = \{ i \in S : T^S(i) = n \} \subseteq S \) and \( B(n) = \{ j \in B : T^B(j) = n \} \subseteq B \) be the set of P2P producers and consumers connected to bus \( n \) with \( \Xi^S_n : S(n) \rightarrow \mathcal{Z}(n) \) and \( \Xi^B_n : B(n) \rightarrow \mathcal{Z}(n) \) be the functions that map them to a unique set of P2P participants \( \mathcal{Z}(n) \) respectively. Further, let \( e^P_n, e^Q_n \in \mathbb{R}^{\vert \mathcal{Z}(n) \vert} \) be the P2P active and reactive power injection vectors with the respective bid price vectors \( \Lambda^P_n, \Lambda^Q_n \in \mathbb{R}^+_{\vert \mathcal{Z}(n) \vert} \). Here, \( e^P_n \) and \( e^Q_n \) couple the nodal OPF problem solved by nodal agent \( n \) (henceforth, termed as NA-\( n \)) with P2P producer optimization problems.

Nodal OPF problem solved by NA-\( n \) can be formulated as follows. Equation (1) is the objective function for all \( n \in N \):

\[
\begin{align*}
O_n^{\text{NA}} & = J^{\text{P}}_n(p^G_n) + J^{\text{Q}}_n(q^G_n) + \Phi^T_n \Theta_n + (\Lambda^P_n)^T e^P_n + (\Lambda^Q_n)^T e^Q_n + \frac{\rho}{2} \left\| \Theta_n - \hat{\Theta}_n \right\|_2^2 + \frac{\rho}{2} \left\| e^P_n - \hat{e}^P_n \right\|_2^2 + \frac{\rho}{2} \left\| e^Q_n - \hat{e}^Q_n \right\|_2^2 \end{align*}
\]

where \( J^{\text{P}}_n : \mathbb{R}^{\vert G(n) \vert} \rightarrow \mathbb{R}_+ \) and \( J^{\text{Q}}_n : \mathbb{R}^+_{\vert G(n) \vert} \rightarrow \mathbb{R}_+ \) are the convex active and reactive power generation cost functions of the set of generators \( \mathcal{G}(n) \) at bus \( n \) respectively. System variables relevant to bus \( n \in N \) are summarized as \( \Theta_n = \begin{bmatrix} w_{A(n)}; w_n; \ell_n; (\ell_m)^T_{m \in \mathcal{C}(n)}; P_n; (P_m)^T_{m \in \mathcal{C}(n)}; Q_n; (Q_m)^T_{m \in \mathcal{C}(n)} \end{bmatrix}^T \in \mathbb{R}^{5+3\vert \mathcal{C}(n) \vert} \), which will be optimized by NA-\( n \) in (5) while considering the corresponding bid price vector \( \Phi_n = \begin{bmatrix} \Phi^\text{\text{F}}_{A(n)}; \Phi^\text{\text{F}}_n; \Phi^\text{\text{F}}_{m \in \mathcal{C}(n)}; \Phi^\text{\text{P}}_n; (\Phi^\text{\text{P}}_m)^T_{m \in \mathcal{C}(n)}; \Phi^\text{\text{Q}}_n; (\Phi^\text{\text{Q}}_m)^T_{m \in \mathcal{C}(n)} \end{bmatrix}^T \). Therein, the nodal OPF problem solved by NA-\( n \) is coupled via \( \Theta_n \) with the nodal OPF problems solved by each NA-\( m \) where \( m \in \mathcal{A}(n) \cap \mathcal{C}(n) \), that manage the physically connected neighbouring buses of bus \( n \). \( \hat{\Theta}_n, \hat{e}^P_n, \) and \( \hat{e}^Q_n \) are the predetermined fixed values of the respective coupling variables, which will be iteratively updated based on the negotiations between the market agents (details are discussed in Section III). Further, the objective function (1) is augmented with quadratic penalty terms that drive the optimal solution towards the fixed values of coupling variables where \( \rho \) is a predefined ADMM parameter [41].
Nodal OPF problem solved by NA-n is subjected to the following grid constraints derived based on the second-order cone model [40]. Henceforth, the dual variables are listed after a colon on the right side of the respective constraints with the same dimensions.

Variable bounds are enforced in (2) for all $n \in N$:
\begin{align}
  w_n & \leq w_n \leq \bar{w}_n; \quad \mu^V_n, \bar{\mu}^V_n, \quad 0 \leq \ell_n \leq \mu^\ell_n \quad (2a) \\
  p^G_n & \leq p^G_n \leq \bar{p}^G_n; \quad e^F_n, \bar{e}^F_n, \quad q^Q_n \leq q^Q_n \leq \bar{q}^Q_n; \quad \bar{\mu}^Q_n, \bar{\mu}^Q_n \quad (2b)
\end{align}

Nodal active and reactive power balance is satisfied by (3) for all $n \in N$:
\begin{align}
  p_n = 1_{\bar{g}(n)}^T \bar{g}_n - p_n^D, \quad q_n = 1_{\bar{q}(n)}^T \bar{q}_n - q_n^D \quad (3a) \\
  \sum_{m \in \mathcal{C}(n)} \left[ P_m + r_m \ell_m \right] - P_n - 1_{\bar{z}(n)}^T e_n^p - p_n = 0 : \varphi_n^p \quad (3b) \\
  \sum_{m \in \mathcal{C}(n)} \left[ Q_m + x_m \ell_m \right] - Q_n - 1_{\bar{z}(n)}^T e_n^q - q_n = 0 : \varphi_n^q \quad (3c)
\end{align}

Therefore, optimizing $e_n^p$ and $e_n^q$ as nodal injections in (3) (instead of fixing as in [16], [17]) while penalizing their deviations from the fixed values in (1), inherently ensures their satisfaction of grid constraints.

For all $n \in N \setminus \{1\}$, (4a)-(4b) respect the Ohm’s Law and relate the line flows and the voltage magnitudes, and (4c)-(4d) respect the (bidirectional) apparent power flow capacity over the distribution line between bus $A(n)$ and bus $n$. Therein, the second-order cone constraint (4b) captures the nonlinearity in power flow and it must be tight (or active) to ensure the exactness of the second-order cone relaxation [42]. Further, the OPF problem of NA-1 is not subjected to (4) as it does not have an ancestor bus.
\begin{align}
  w_{A(n)} = w_n + (r_n^2 + x_n^2) \ell_n + 2r_n P_n + 2x_n Q_n : \varphi_n^V \\
  P_n + Q_n^2 \leq \ell_n^2 \quad (4b) \\
  P_n + Q_n^2 \leq \bar{F}_n^2 : \bar{\mu}_n^F \\
  \left[ P_n + r_n \ell_n \right]^2 + \left[ Q_n + x_n \ell_n \right]^2 \leq \bar{F}_n^2 : \bar{\mu}_n^F \quad (4d)
\end{align}

To this end, the nodal OPF problem solved by NA-n can be summarized as where $n \in N$ can be as follows:
\begin{align}
  \min_{e_n^p, e_n^q, T_n, \Theta_n, e_n^S} & \mathcal{O}^N_n \quad \text{s. t. (2)-(4)} \quad (5)
\end{align}

C. Producer and Consumer Optimization Models in the P2P Energy Market

The objective functions $\mathcal{O}_i^S : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^{[S]} \to \mathbb{R}$ of producer $i \in S$ and $\mathcal{O}_j^B : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^{[S]} \to \mathbb{R}$ of consumer $j \in S$ are defined below to achieve the optimal cost and welfare for producers and consumers respectively.
\begin{align}
  \mathcal{O}_i^S = H_i(e^S_{i,j} - \Lambda_{i,j}^S, e^S_{i,j} - \Lambda_{i,j}^S, e^S_{i,j} - \Pi_{i,j}^S) + \frac{\rho}{2} \left[ \left( e^S_{i,j} - e^S_{i,j} \right)^2 + \left( e^S_{i,j} - e^Q_{i,j} \right)^2 + \left\| E_i^S : - \hat{E}_i \right\|_2^2 \right) \quad (6a) \\
  \mathcal{O}_j^B = -H_j(e^B_{j,i} - \Lambda_{j,i}^B, e^B_{j,i} - \Lambda_{j,i}^B, e^B_{j,i} - \Pi_{j,i}^B) + \frac{\rho}{2} \left[ \left( e^B_{j,i} - e^B_{j,i} \right)^2 + \left( e^B_{j,i} - e^Q_{j,i} \right)^2 + \left\| E_j^B : - \hat{E}_j \right\|_2^2 \right) \quad (6b)
\end{align}

where $H_i : \mathbb{R}_+ \to \mathbb{R}$ is the convex cost function of producer $i \in S$ and $H_j : \mathbb{R}_+ \to \mathbb{R}$ is the concave welfare function of consumer $j \in B$. These two functions take the general shapes as explained in [7, Sec. II-A]. $e^S_{i,j} \in \mathbb{R}_+^{[S]}$ and $e^Q_{i,j} \in \mathbb{R}_+^{[S]}$ are the total active and reactive power production vectors of producers; $e^B_{j,i} \in \mathbb{R}_+^{[S]}$ and $e^Q_{j,i} \in \mathbb{R}_+^{[S]}$ are the total active and reactive power consumption vectors of consumers; and $E^S_i, E^B_j \in \mathbb{R}_+^{[S]}$ are the energy transaction matrices of producers and consumers respectively. $E^S_i, E^B_j$ is the energy sold by producer $i$ to consumer $j$. $E^B_{j,i}$ is the energy purchased by consumer $j$ from producer $i$. It should be noted that the compliance of energy transactions between P2P agents requires $E^S = E^B$. Hence, the two matrix variables $E^S_i$ and $E^B_{j,i}$ couple the optimization problems (7) and (8) solved by the P2P trading pair $(i, j)$ separately. Similarly, $e^S_{i,j}, e^Q_{i,j}$ couple the P2P market and the ancillary services market (i.e., defined in (5) as nodal OPF problems). $\Lambda_{i,j}^S \in \mathbb{R}_+^{[S]}$, $\Pi_{i,j}^S \in \mathbb{R}_+^{[S]} \times [B]$, $\bar{\Lambda}_{i,j}^B \in \mathbb{R}_+^{[B]}$, $\bar{\Pi}_{i,j}^B \in \mathbb{R}_+^{[B]} \times [S]$, and $\bar{\Lambda}_{i,j}^B \in \mathbb{R}_+^{[B]} \times [S]$ are the bid prices of the respective coupling variables. $e^S_{i,j}, e^B_{j,i}, e^Q_{i,j}$, $E^S$, and $E^B$ are the predetermined fixed values of the respective coupling variables. To this end, the optimization problem solved by P2P seller agent $i \in S$ can be formulated as follows:
\begin{align}
  \min_{e^S_{i,j}, e^Q_{i,j}, e^B_{j,i}} & \mathcal{O}_i^S \quad \text{s. t. (7a)-(7b)} \\
  \text{s. t.} & e^S_{i,j} - E^B_{j,i} \mathbf{1}_{[S]} = 0 : \varphi^S_i \\
  & E_i^S : \geq 0 : \Omega_i^S \\
  & e^S_{i,j} \leq e^S_{i,j} : e^S_{i,j}, \bar{\mu}^S_{i,j} \quad (7d) \\
  & - \bar{\delta}_i, e^S_{i,j} \leq e^Q_{i,j}, \bar{\mu}^Q_{i,j} \quad (7e)
\end{align}

Power balance at each producer is satisfied by (7b). Non-negativity of supply requests is ensured by (7c). The total active and reactive power production limits are respected by (7d)-(7e). The sellers in the P2P market, such as solar PV, ESSs (in discharging mode), etc., are inverter-interfaced devices that are capable of supplying reactive power to the distribution grid [43]. The reactive power generation is limited by the maximum permissible leading and lagging power factors, which is imposed via $\delta^S_i$ and $\delta^Q_i$ respectively. In case if seller $i \in S$ operates at a constant power factor, which will be imposed with $\bar{\delta}^S_i = -\bar{\delta}^S_i = \sigma^S_i = \sigma^Q_i$ in the simulations.

The optimization problem solved by P2P buyer agent $j \in B$ can be formulated as follows:
\begin{align}
  \min_{e^B_{j,i}, e^Q_{j,i}, e^S_{i,j}} & \mathcal{O}_j^B \quad \text{s. t. (8a)-(8b)} \\
  \text{s. t.} & e^B_{j,i} - E^B_{i,j} \mathbf{1}_{[S]} = 0 : \varphi^B_j \\
  & E_j^B : \geq 0 : \Omega_j^B \\
  & e^B_{j,i} \leq e^B_{j,i} : e^B_{j,i}, \bar{\mu}^B_{j,i} \quad (8d) \\
  & - \bar{\delta}_i^{B}, e^B_{j,i} \leq e^Q_{j,i}, \bar{\mu}^Q_{j,i} \quad (8e)
\end{align}

Power balance at each consumer is satisfied by (8b). Non-negativity of demand requests is ensured by (8c). The total active and reactive power consumption limits are respected by (8d)-(8e). Some of the buyers in the P2P market, such as ESSs (in charging mode), are also inverter-interfaced devices
that are capable of supplying reactive power to the distribution grid. The maximum permissible leading and lagging power factors of those devices are imposed by \( \delta_B \) and \( \delta^B \) respectively. In case if buyer \( j \in B \) is a displaceable consumer, the power factor can be constant and lagging, which will be imposed with \( \delta^B = -\delta^B = \sigma^B \) in the simulations.

III. COORDINATION BETWEEN AGENTS AND OPTIMAL OPERATION OF MARKET PARTICIPANTS

A. ADMM-based Coordination and Optimization

The coordination between the coupling variables of (5), (7), and (8) towards the optimal market clearing is achieved based on the ADMM [41]. Therein, NA-n for all \( n \in N \) runs Algorithm 1, while the P2P producer \( i \) for all \( i \in S \) and the P2P consumer \( j \) for all \( j \in B \) run Algorithm 2 in parallel.

Algorithm 1: Optimization and Coordination of NA-n

Input : \( \Phi_n(0), \Lambda_n^P(0), \Lambda_n^Q(0), \Theta_n(0), e^P_n(0), e^Q_n(0), \varepsilon_\Theta, \varepsilon_e, k = 0 \)

1 while \( \| \Delta e_n(k) \|_\infty \geq \varepsilon_e \) and \( \| \Delta \Theta_n(k) \|_\infty \geq \varepsilon_\Theta \) do

2 Execute (5): \( x \)-update.

3 Send information to NA-A(n). Receive information from NA-m; \( \forall m \in C(n) \).

4 Exchange information with set of P2P agents \( Z(n) \).

5 Update the fixed coupling variables as in (9)-(10).

6 Send information to NA-m; \( \forall m \in C(n) \). Receive information from NA-A(n).

7 Update the bid prices as in (11).

8 \( k \leftarrow k + 1 \)

9 end

Output: \( \Phi_n, \Lambda_n^P, \Lambda_n^Q, \Theta_n, e^P_n, e^Q_n \)

Algorithm 2: Optimization and Coordination of Producer i/Consumer j

Input : \( \Pi_i, \Lambda_i^P(0), \Lambda_i^Q(0), \Theta_i(0), e^P_i(0), e^Q_i(0)/ \Pi_j, \Lambda_j^P(0), \Lambda_j^Q(0), \Theta_j(0), e^P_j(0), e^Q_j(0)/\varepsilon_E, \varepsilon_e, k = 0 \)

1 while \( \| \Delta e(k) \|_\infty \geq \varepsilon_e \) and \( \| \Delta E(k) \|_\infty \geq \varepsilon_E \) do

2 Execute (7)/(8): \( z \)-update.

3 Exchange information with NA-I^S(i)/NA-I^B(j)

4 Exchange \( E_i, E_j \) with \( B/S \) respectively.

5 Update the fixed coupling variables as in (13).

6 Update the bid prices as in (14)/(15).

7 \( k \leftarrow k + 1 \)

8 end

Output: \( \Pi_i^*, \Lambda_i^P, \Lambda_i^Q, E_i^*, \Pi_j^*, \Lambda_j^P, \Lambda_j^Q, E_j^*, E_i, E_j \)

As summarized in Algorithm 2 at iteration \( k \), kodal agents execute the nodal OPF problem (5) in Step 2. Let \( X^n \) be the coupling variable \( x \) computed by NA-n. In Step 3, NA-n sends \( \hat{w}^m_{A(n)} \), \( e^m_n \), \( P^m_n \), and \( Q^m_n \) to NA-A(n). Accordingly, NA-n receives \( w_{A(m)}^m, \hat{e}_m^m, P^m_n, \) and \( Q^m_n \) from NA-m for all \( m \in C(n) \). Then, NA-n exchanges \( e^P_n \) and \( e^Q_n \) with the set of P2P agents \( Z(n) \) as in Step 4. In Step 5, NA-n updates the fixed coupling variable values \( \hat{w}^m_{A(n)}, \hat{e}_m^m, P^m_n \), and \( Q^m_n \) as in (9), and sends them to all NA-m where \( m \in C(n) \) in Step 6.

\[
\hat{w}^m = \frac{1}{1 + |C(n)|} \left[ w_{m,n}^n + \sum_{m \in C(n)} w_{m,A(n)}^m \right], \hat{e}_m = \frac{1}{2} \left[ \hat{e}_m^m + \hat{e}_m^m \right], P^m_n = \frac{1}{2} \left[ P^m_n + P^m_n \right], Q^m_n = \frac{1}{2} \left[ Q^m_n + Q^m_n \right]
\]

Further, \( \hat{e}^P_n \) and \( \hat{e}^Q_n \) are updated as in (10).

\[
\hat{e}^P_n = \frac{1}{2} \left[ \hat{e}^P_n + \hat{e}^P_n \right], \hat{e}^Q_n = \frac{1}{2} \left[ \hat{e}^Q_n + \hat{e}^Q_n \right]
\]

where \( e^P_n = \left[ (e^P_{i,j} + e^P_{i,n}) \right] \) and \( e^Q_n = \left[ (e^Q_{i,j} + e^Q_{i,n}) \right] \)
For all $j \in B$:

$$
\Pi^B_{ij}(k+1) = \Pi^B_{ij}(k) + \rho [E^S_{ij}(k) - \hat{E}^S_{ij}(k)] \\
\Lambda^B_{ij}(k+1) = \Lambda^B_{ij}(k) + \rho [\mathbf{e}^B_{ij}(k) - \hat{\mathbf{e}}^B_{ij}(k)] \\
\Lambda^Q_{ij}(k+1) = \Lambda^Q_{ij}(k) + \rho [e^B_{ij}(k) - \hat{e}^B_{ij}(k)]
$$

Algorithm 2 terminates when the metrics of residuals defined in (16a)/(16b) reach the tolerance levels specified in Step 1 [41].

$$
\Delta e(k) = [e^S_{ij}(k) - \hat{e}^S_{ij}(k), e^Q_{ij}(k) - \hat{e}^Q_{ij}(k)]^T, \\
\Delta E(k) = E^S_{ij}(k) - \hat{E}^S_{ij}(k); \forall i \in S \\
\Delta e(k) = [e^B_{ij}(k) - \hat{e}^B_{ij}(k), e^Q_{ij}(k) - \hat{e}^B_{ij}(k)]^T, \\
\Delta E(k) = E^B_{ij}(k) - \hat{E}^B_{ij}(k); \forall j \in B
$$

To this end, the key steps of the algorithms executed by nodal agents and prosumers in the proposed decentralized market mechanism along with their mutual interactions are concisely illustrated in Fig. 3.

![Fig. 3. Algorithms executed by nodal agents and prosumers within the decentralized market mechanism.](image)

It should be noted that none of the P2P agents in the proposed decentralized market framework are required to be aware of any specific information such as DLMPs, loss sensitivity factors, power transfer distribution factors, etc., of the other P2P agents for the market clearing process, apart from the trading price and energy trading amounts. Hence, the P2P agents can participate with anonymous IDs in each market interval such that their trading price or energy trading amount preferences cannot be recorded for making predictions in future. Further, the proposed decentralized market framework also allows a P2P agent to represent with two anonymous IDs (if preferred); one for the negotiations between other P2P agents; and the other for the negotiations with the corresponding nodal agent. Therefore, the nodal agent is also unaware of the prosumers identity during the market clearing process which enhances the fairness of energy trading. As such, the proposed decentralized market framework preserves fairness of the market mechanism, prosumer autonomy, and prosumer anonymity without compromising the market efficiency.

B. Proof of Convergence of the Market Clearing Mechanism

In the following, the conditions for convergence of Algorithms 1 and 2, and the global optimality of the proposed market framework are discussed.

**Assumption 1:** Objective functions $J^P_n, J^Q_n; \forall n \in N, H_i; \forall i \in S$, and $-U_j; \forall j \in B$ are closed, proper, and convex real-valued functions.

**Assumption 2:** There exists a strictly feasible solution for the overall optimization problem, i.e., the co-optimization of P2P energy trading enabled OPF for the distribution grid, for which (3), (4a)–(4b), (7a), and (8a) satisfy, and (2), (4c)–(4d), (7c)–(7e), and (8c)–(8e) hold with strict inequality.

It should be noted that Assumption 1 is a requirement to be satisfied by the market participants not only to support the market equilibrium but also to maximize their social welfare. Assumption 2 is a requirement for the operability of the distribution grid, i.e., there are several feasible (non-optimal) operating points at which the power balance is maintained without reaching the technical or supply capability limitations.

**Remark 1:** By Assumption 1 and by definition of (1); $\forall n \in N, (6a); \forall i \in S$, and (6b); $\forall j \in B$, the objective functions $O^{NA}_n; \forall n \in N, O^P; \forall i \in S$, and $O^P; \forall j \in B$ are closed, proper, and convex real-valued functions.

**Remark 2:** By construction, (5) $\forall n \in N, (7) \forall i \in S, (8) \forall j \in B$, and hence the overall optimization problem constitutes convex inequality constraints and affine equality constraints.

The properties stated in Assumption 2, and Remarks 1 and 2 jointly satisfies the Slater’s condition for strong duality discussed in [44, Ch. 5.9]. This implies that the overall optimization problem constitutes a saddle point and holds zero duality gap. Hence, the proposed market framework satisfies the necessary and sufficient conditions stated under Assumptions 1 and 2 in Chapter 3.2 of [41] for convergence and global optimality of ADMM-based distributed optimization algorithms. The proof based on the general structure of ADMM can be found in [41, Appendix A]. Therefore, the iterates of Algorithms 1 and 2 satisfy residual convergence, objective convergence, and dual variable (i.e., bid price and DLMP) convergence as $k \to \infty$ [41, Ch. 3.2.1]. As described in [42, Sec. V-C], the second-order cone relaxation introduced in (5) always results in valid optimal OPF solutions with tight (or active) (4b), provided that the network topology is connected and radial, and there are no upper bounds on loads. The first condition is generally satisfied for distribution grids. The second condition can be incorporated by relaxing the equality constraints in (3a) to inequality constraints, i.e., $\leq$. However, in most of the cases, it would not change the optimum as more generation to over satisfy the loads at any node will unnecessarily increase the respective objective function $O^{NA}_n$ for all $n \in N$, the optimization solvers always tend to hold (3a) tight. Hence, the solution of Algorithm 1 is (expected to be) a valid global optima for the OPF problem which addresses the resource utilization of the distribution grid.

C. Price Settlement between Market Participants

By substituting $\hat{E}^S_{ij}$ in (13b) into (14a) and (15a), the following can be obtained for all $i \in S$ and $j \in B$.

$$
\Pi^S_{ij}(k+1) = \Pi^S_{ij}(k) - \frac{\rho}{2} [E^S_{ij}(k) - \hat{E}^S_{ij}(k)] \\
\Pi^B_{ij}(k+1) = \Pi^B_{ij}(k) + \frac{\rho}{2} [e^B_{ij}(k) - \hat{e}^B_{ij}(k)]
$$

They lead to the following recursive formula.

$$
\Pi^S(k+1) - \Pi^S(k) = \Pi^S(k) - \Pi^B(k)
$$
Hence, if Algorithm 2 is initiated with $\Pi_i^S(0) = \Pi_i^B(0)$, then

$$\Pi_i^S(k) = \Pi_i^B(k); \forall k \in \{0, 1, 2, \ldots\} \quad (19)$$

Corollary, in each iteration $k$ of Algorithms 1 and 2, the following relations between the updated bid prices corresponding to the coupling variables also hold.

$$\Phi_{n,[m]} = \sum_{m \in \mathcal{C}(n)} \Phi_{A(m)}^c, \forall n \in \{n \in \mathcal{N} : \mathcal{C}(n) \neq \emptyset\} \quad (20a)$$

$$\Phi_{n,[A(n)]}^c = -\Phi_{n,[m]}^c, \forall n \in \mathcal{N} \setminus \{1\} \quad (20b)$$

$$\Lambda_{P,i} = \Lambda_{n,z}^P, \Lambda_{B,i}^P = \Lambda_{n,z}^B; \forall i \in S, n \in \mathcal{B}(i), \pi = \Xi_n(i) \quad (20c)$$

$$\Lambda_{P,j} = \Lambda_{n,z}^P, \Lambda_{B,j}^P = \Lambda_{n,z}^B; \forall j \in \mathcal{B}, n \in \mathcal{B}(j), \pi = \Xi_n(j) \quad (20d)$$

To this end, each NA-$m \forall m \in \mathcal{C}(n)$ trades with NA-$n$ to maintain a coupled ancestor bus voltage $w_{A(m)} = w_{A(n)}$ with a combined bid price as in (20a). Hence, the bid price $\Phi_{A(m)}^c$ offered from each NA-$m$ to NA-$n$ is a trade-off among them for the desired voltage which minimizes their local cost. As in (20b), each NA-$n$ trades with NA-$A(n)$ for the desired current flow $\ell_{A(n)} = \ell_{n,m}^A$, active power flow $P_{n,A(n)} = P_{n,m}$, and reactive power flow $Q_{n,A(n)} = Q_{n,m}$. Hence, each NA-$n \forall n \in \mathcal{N}$ will pay $\Phi_{A(n)}^c w_{A(n)} + \Phi_{P,n}^c P_{n}^c + \Phi_{Q,n}^c Q_{n}^c$ to NA-$A(n)$ upon market equilibrium. Further, consumer $j \forall j \in \mathcal{B}$ will pay $\Lambda_{P,j}^B e_{P,j}^B + \Lambda_{B,j}^B e_{B,j}^B$ to NA-$\mathcal{T}(j)$, producer $i \forall i \in \mathcal{S}$ will receive $\Lambda_{P,i}^P e_{P,i}^P + \Lambda_{Q,i}^P e_{Q,i}^P$ from NA-$\mathcal{T}(i)$, and consumer $j$ will pay $\Pi_{i,j}^B e_{i,j}^B = (\Pi_{i,j}^S e_{i,j}^S)$ to producer $i$.

IV. P2P ENERGY MARKET EQUILIBRIUM ANALYSIS

The optimal solution of (7) for all $i \in \mathcal{S}$ will always satisfy the following three Karush–Kuhn–Tucker (KKT) conditions.

$$-\Pi_{i,j}^S + \rho(E_{i,j}^S - \hat{E}_{i,j}) - \varphi_i^S - \Omega_{i,j}^S = 0; \forall j \in \mathcal{B} \quad (21a)$$

$$\partial \mathcal{H}_i(e_{P,i}^S) = -\delta_i^S e_{P,i}^S + \rho(e_{P,i}^S - e_{P,i}^S) + \varphi_i^S + \mu_i^S - \bar{\mu}_i^S \quad (21b)$$

$$\partial \mathcal{H}_i(e_{Q,i}^S) = -\delta_i^S e_{Q,i}^S + \rho(e_{Q,i}^S - e_{Q,i}^S) + \mu_i^S - \bar{\mu}_i^Q \quad (21c)$$

Solving (21) subject to termination criteria (16a) $\forall i \in \mathcal{S}$, while assuming (7d) is strict, and $E_{i,j}^S$ is non-binding deduces

$$\Pi_{i,j}^S = \frac{\partial \mathcal{H}_i(e_{P,i}^S)}{\partial e_{P,i}} - \delta_i^S e_{P,i}^S + \Psi_{Q,i}^S; \forall j \in \mathcal{B} \quad (22)$$

where

$$\Psi_{Q,i}^S = \left\{\begin{array}{ll}
-\delta_i^S \Lambda_{Q,i}^S & \text{if } e_{Q,i}^S = \sigma_i^S e_{P,i}^S \\
\delta_i^S \Lambda_{Q,i}^S & \text{if } e_{Q,i}^S = -\sigma_i^S e_{P,i}^S \\
\end{array}\right. \quad (23)$$

Similarly, the optimal solution of (8) for all $j \in \mathcal{B}$ will always satisfy the following three KKT conditions.

$$\Pi_{i,j}^B + \rho(E_{i,j}^B - \hat{E}_{i,j}) - \varphi_j^B - \Omega_{i,j}^B = 0; \forall i \in \mathcal{S} \quad (24a)$$

$$\partial \mathcal{H}_j(e_{P,j}^B) = \Lambda_{P,j}^B + \rho(e_{P,j}^B - e_{P,j}^B) + \varphi_j^B + \mu_j^B - \bar{\mu}_j^B \quad (24b)$$

$$\Lambda_{Q,j}^B + \rho(e_{Q,j}^B - e_{Q,j}^B) + \mu_j^B - \bar{\mu}_j^Q = 0 \quad (24c)$$

Solving (24) subject to termination criteria (16b) $\forall j \in \mathcal{B}$, while assuming (8d) is strict, and $E_{i,j}^B$ is non-binding, deduces

$$\Pi_{i,j}^B = \frac{\partial \mathcal{H}_j(e_{P,j}^B)}{\partial e_{P,j}} - \delta_j^B e_{P,j}^B - \Psi_{Q,j}^B; \forall i \in \mathcal{S} \quad (25)$$

where

$$\Psi_{Q,j}^B = \left\{\begin{array}{ll}
-\delta_j^B \Lambda_{Q,j}^B & \text{if } e_{Q,j}^B = \sigma_j^B e_{P,j}^B \\
\delta_j^B \Lambda_{Q,j}^B & \text{if } e_{Q,j}^B = -\sigma_j^B e_{P,j}^B \\
\end{array}\right. \quad (26)$$

Since, $\Pi_{i,j}^S = \Pi_{i,j}^B$ as proven in (19), the following can be deduced for all $i \in \mathcal{S}$ and $j \in \mathcal{B}$

$$\Gamma_{i,j}^P - \Lambda_{B,i}^B + \delta_i^B \Lambda_{Q,i}^B = \Psi_{Q,i}^B \quad (27)$$

KKT conditions satisfied by the optimal solution of (5) ensure $\Lambda_{P,i}^S = \mathcal{I}(\varphi_{P_i}^S)$, and $\Lambda_{Q,i}^S = \mathcal{I}(\varphi_{Q_i}^S)$ for bus $n \in \mathcal{N}$. Then, with (20c)-(20d), $\Lambda_{P,i}^B = \mathcal{I}(\varphi_{P_i}^S)$ and $\Lambda_{Q,i}^B = \mathcal{I}(\varphi_{Q_i}^S)$ for consumer $j \in \mathcal{B}$, and $\Lambda_{P,i}^B = \mathcal{I}(\varphi_{P_i}^B)$ and $\Lambda_{Q,i}^B = \mathcal{I}(\varphi_{Q_i}^B)$ for producer $i \in \mathcal{S}$ also hold. By definition, DLMF-PQ, i.e., $\varphi_{P_i}^S/\varphi_{Q_i}^S$, at bus $n$ is the marginal increase in sum of the active/reactive power generation cost at the root node and the cost of supporting ancillary services caused by an increase of active/reactive power demand at bus $n$, respectively [17], [45].

In this respect, $\Gamma_{i,j}^P = \Lambda_{B,i}^B - \Lambda_{B,i}^S = \varphi_{P_i}^B - \varphi_{P_i}^S$, reflects the marginal cost/reward of supporting ancillary services associated with a P2P active power transaction from bus $\mathcal{T}(i)$ to bus $\mathcal{T}(j)$ in the distribution grid [17]. Similarly, $\Gamma_{i,j}^Q$ reflects the marginal cost/reward of supporting ancillary services associated with the reactive power injections caused by a P2P active power transaction from bus $\mathcal{T}(i)$ to bus $\mathcal{T}(j)$ in the distribution grid. It should be noted that $\Gamma_{Q,i}^S$ is negative if both producer $i$ and consumer $j$ can supply reactive power as per (23) and (26). As such, the inverter-interfaced devices are rewarded depending on their support for ancillary services. Moreover, $\Gamma_{i,j}^P + \Gamma_{i,j}^Q$ in (27) reflects the marginal cost/reward of supporting ancillary services associated with the P2P active power transaction $E_{P,i,j}^S$ (or $E_{P,i,j}^B$). Hence, when $e_{P,i,j}^S$ and $e_{P,i,j}^B$ are within their capability limits, the P2P active power transactions are facilitated only when the difference between the marginal welfare of consumer $j$ and the marginal cost of producer $i$ is sufficiently higher than $\Gamma_{i,j}^P + \Gamma_{i,j}^Q$.
market framework accurately allocates the costs/rewards of supporting ancillary services associated with each P2P active power transaction to the respective producer-consumer pairs. As such, the proposed market framework provides a fair and equal platform for P2P energy trading without compromising the efficiency and feasibility of the distribution grid operations.

As an increment of \( E_{ij}^p \) and \( E_{ij}^q \) increases \( \overline{c}_{ij}^p \) and \( \overline{c}_{ij}^q \), respectively, Fig. 4 interprets the three scenarios I, II, and III that market equilibrium exists between producer-consumer pairs depending on \( \Gamma_{ij}^p + \Gamma_{ij}^q \). This illustrates that reducing \( \Gamma_{ij}^p + \Gamma_{ij}^q \) via the benefits of reactive power support enhances the amount of energy sharing between the respective producer-consumer pairs.

![Fig. 4. Market clearing price and energy trading amount variation between producer and consumer pairs](image)

**V. RESULTS AND DISCUSSION**

**A. Simulation Settings**

Case studies are conducted on the IEEE 33-bus system which has a total static demands of 3.715 MW and 2.3 Mvar [46]. The static demands are doubled; voltage bounds are set to [0.9, 1.1] p.u.; and the transfer capacity of lines are set to 5.0 MVA to illustrate the effectiveness of the proposed market framework. The participants of the P2P market and the ancillary service market are documented in Tables I and II respectively. Their spatial distribution over the IEEE 33-bus system is depicted in Fig. 5. In addition, \( \delta_i^p = \delta_i^q = 0.4 \) \( \forall i \in S \); \( \delta_j^p = \delta_j^q = 0.4 \) \( \forall j \in \{1, 5\} \); and \( \delta_j^p = \delta_j^q = 0.3 \) \( \forall j \in \{2, 3, 4, 6, 7\} \). The ADMM parameter \( \rho = 5.0 \); \( \varepsilon_\theta = 10^{-4} \); and \( \varepsilon_E = 10^{-5} \). Algorithms 1 and 2, and the optimization problems (5), (7), and (8) were programmed in MATLAB and solved using Gurobi [47]. All the simulations were performed on a desktop PC with an Intel® Core i7-4770U four-core CPU processor running at 3.40 GHz with 8 GB of RAM.

<table>
<thead>
<tr>
<th>Bus Index</th>
<th>P0</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE I**

<table>
<thead>
<tr>
<th>Bus Index</th>
<th>P2P Market Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
</tr>
</tbody>
</table>

**Fig. 5. Schematic diagram of the IEEE 33-bus system, including the spatial distribution of producers and ancillary service providers.**

**B. Efficiency Comparison**

To illustrate the impact of P2P power injections on the power loss, voltage, and congestion management of the distribution grid, simulations were conducted under four cases.

- Case 1: Nodal OPF without considering the P2P energy trading.
- Case 2: P2P energy trading without considering grid interactions, and Nodal OPF.
- Case 3: The proposed market framework (Algorithms 1 and 2 are executed) without considering the reactive power capabilities/demands of the P2P participants.
- Case 4: The proposed market framework (Algorithms 1 and 2 are executed) while considering the reactive power capabilities/demands of the P2P participants.

As illustrated in Fig. 6, production and consumption of the P2P participants are significantly different when grid interactions are considered. As reported in Table III, the P2P injections computed in Case 2 resulted in an 11.44% increase in the total operating cost of the distribution grid (compared to Case 1). Hence, P2P trading enabled distribution operation requires efficient algorithms for optimum utilization of resources. In contrast, the optimal solutions under Cases 3 and 4 acceptably converged to those of their optimal centralized counterparts with a relative optimality gap \( \ll 10^{-3} \). Hence, the proposed market framework has the potential to guarantee grid constraint satisfaction and global optimality for P2P trading enabled distribution system operation. Table III reports that by enabling P2P trading, the total operating cost of the distribution grid/nodal agents has been reduced by 3.03% in Case 3, and by enabling their reactive power integration, it has been further reduced up to 4.87% in Case 4. Although the total welfare of the P2P participants has been compromised from Case 2 to Case 3 for grid constraint feasibility, it has been improved in Case 4 with the rewards offered for supporting reactive power.

![Fig. 6. Production and consumption of P2P participants](image)
improved in Case 4 compared to Case 3. Table IV reports the power transactions between the participants in the P2P market under Case 4.

### TABLE III

<table>
<thead>
<tr>
<th>Case</th>
<th>Dist. Grid/nodal agents</th>
<th>P2P Producers</th>
<th>P2P Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160.05 **</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>2</td>
<td>179.03 **</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>3</td>
<td>155.79 −12.34</td>
<td>−13.30</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>152.83 −12.99</td>
<td>−16.55</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Production and consumption of producers (P1-P6) and consumers (C1-C7) respectively for Cases 2-4.

![Fig. 6. Production and consumption of producers (P1-P6) and consumers (C1-C7) respectively for Cases 2-4.](image)

Fig. 7. Comparison of (a) DLMP-P, (b) DLMP-Q, and (c) voltage magnitudes under Cases 1-4 respectively.

![Fig. 7. Comparison of (a) DLMP-P, (b) DLMP-Q, and (c) voltage magnitudes under Cases 1-4 respectively.](image)

### TABLE IV

<table>
<thead>
<tr>
<th>P2P Power Transactions ((E^{S*} = E^{B*}))</th>
<th>[MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1)</td>
<td>0.2081</td>
</tr>
<tr>
<td>(P_2)</td>
<td>0.7029</td>
</tr>
<tr>
<td>(P_3)</td>
<td>0.0142</td>
</tr>
<tr>
<td>(P_4)</td>
<td>0.6525</td>
</tr>
<tr>
<td>(P_5)</td>
<td>0.4625</td>
</tr>
<tr>
<td>(C_1)</td>
<td>0.3021</td>
</tr>
<tr>
<td>(C_2)</td>
<td>0.2477</td>
</tr>
<tr>
<td>(C_3)</td>
<td>0.3459</td>
</tr>
<tr>
<td>(C_4)</td>
<td>0.1645</td>
</tr>
<tr>
<td>(C_5)</td>
<td>0.1954</td>
</tr>
<tr>
<td>(C_6)</td>
<td>0.2639</td>
</tr>
<tr>
<td>(C_7)</td>
<td>0.7029</td>
</tr>
</tbody>
</table>

C. Convergence and Computational Performance

Fig. 8 shows the 2-norms of the residuals of each nodal agent, producer, and consumer evaluated in Algorithms 1 and 2 over the iterations. It can be observed that \(\|\Delta \Theta(k)\|_2 < \sqrt{N} \times 10^{-4}\) p.u., \(\|\Delta e(k)\|_2 < \sqrt{\min\{||S||, ||B||\}} \times 10^{-4}\) p.u., and \(\|\Delta E(k)\|_2 < 10^{-4}\) p.u. when algorithms converge in 585 iterations. That satisfies the acceptable termination criteria for ADMM-based optimization-coordination algorithms as mentioned in [41, Ch. 3.3.1]. Fig. 9 illustrates the maximum computation time taken for Step 1 of Algorithm 1 (\(x\)-update) out of 33 executions of (5) in each iteration, and the maximum computation time taken for Step 1 of Algorithm 2 (\(z\)-update) out of 6 executions of (7) and 7 executions of (8) in each iteration. Hence, the total computation time excluding the communication overhead can be estimated as 5.10 s.

### TABLE V

<table>
<thead>
<tr>
<th>Marginal Costs/Rewards of Ancillary Services: (\Gamma^{p*} + \Gamma^{Q*})</th>
<th>[$/MWhr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1)</td>
<td>−1.94</td>
</tr>
<tr>
<td>(P_2)</td>
<td>−0.496</td>
</tr>
<tr>
<td>(P_3)</td>
<td>−0.120</td>
</tr>
<tr>
<td>(P_4)</td>
<td>−0.427</td>
</tr>
<tr>
<td>(P_5)</td>
<td>−0.120</td>
</tr>
<tr>
<td>(P_6)</td>
<td>−0.427</td>
</tr>
<tr>
<td>(P_7)</td>
<td>−0.120</td>
</tr>
</tbody>
</table>

Fig. 10 shows the trading price decomposition between Consumer 2 and the set of producers. Consumers in P2P energy trading only prefer to trade when the effective marginal costs of the offers (i.e., Data III in Fig. 10) are less than their marginal welfare (i.e., Data IV in Fig. 10). It can be observed that the marginal generation cost (i.e., Data I in Fig. 10) of Producer 4 is lesser than that of Producers 2 and 6. However, due to the negative ancillary service costs (i.e., Data II in Fig. 10) between Consumer 2 and each of them compared to the significant ancillary service costs between Consumer 2 and Producer 4 forces Consumer 2 to purchase energy from Producers 2 and 6, and abandon Producer 4. This consumer behaviour is an illustration of the analysis provided in Fig. 4 in Section IV. As such, the proposed market framework promotes P2P participants to match trading pairs that reduces power losses, enhances voltage, and supports congestion management of the distribution grid.

![Fig. 8. Trajectories of the residuals evaluated in Algorithms 1 and 2.](image)

![Fig. 9. Maximum computation time taken for solving optimization problems in two algorithms over the iterations.](image)

D. Market Equilibrium Analysis

Marginal costs/rewards of ancillary services associated between producer-consumer pairs are reported in Table V. Therein, the negative values imply that the P2P power transactions between the respective producer-consumer pair reduce the power losses, or improve the voltage profile, or reduce the network congestion of the distribution system and vice versa.
Fig. 11 shows the trading price decomposition between Producer 1 and the set of consumers. Although Consumers 1, 5, and 6 offer higher marginal welfare, energy transactions between Producer 1 and each of them has been restricted as they have already reached their maximum consumption limits.

Fig. 11 shows the trading price decomposition between Producer 1 and the set of consumers. Although Consumers 1, 5, and 6 offer higher marginal welfare, energy transactions between Producer 1 and each of them has been restricted as they have already reached their maximum consumption limits.

E. Scalability Assessment

To verify the scalability of the proposed market framework, a large-scale test system was constructed by combining five test systems setup in Section V-A. Therein, one test system was considered as the base-system in which the point-of-common-coupling (or root node) is preserved. The points-of-common-coupling of the remaining four test systems were combined with the end nodes, i.e., buses 18, 22, 25, and 33, of the base-system. Hence, the resulting distribution system consists of $|N| = 161$ buses, $|L| = 160$ lines, $|G| = 26$ generation sources including the point-of-common-coupling, and a total static demands of 37.15 MW and 23.0 MVar. The number of P2P producers and consumers were also increased by five times, i.e., $|S| = 30$ and $|B| = 35$. The transfer capacity of lines of the base-system were set to 15.0 MVA each and those of the remaining four systems were unchanged. The ADMM parameter $\rho = 10.0$ and the termination tolerances are unchanged.

Fig. 12 shows the 2-norms of the residuals of each nodal agent, producer, and consumer evaluated in Algorithms 1 and 2 over the iterations. Similar to Fig. 8, it can be observed that $\|\Delta \Theta(k)\|_2 < \sqrt{|N|} \times 10^{-4}$ p.u., $\|\Delta e(k)\|_2 < \sqrt{\min\{|S|, |B|\}} \times 10^{-4}$ p.u., and $\|\Delta E(k)\|_2 < 10^{-4}$ p.u. when algorithms converge in 1,099 iterations. That satisfies the acceptable termination criteria for ADMM-based optimization-coordination algorithms as mentioned in [41, Ch. 3.3.1].

To illustrate the impact of P2P power injections on the power losses, voltage improvement, and congestion management of the distribution grid, the simulations were conducted under the four cases described in Section V-B. Similar to the results in Section V-B, it is reported in Table VI that P2P injections computed in Case 2 resulted in a 6.20% increase in the total operating cost of the distribution grid (compared to Case 1). In contrast, the optimal solutions under Cases 3 and 4 also changed to those of their optimal centralized counterparts with a relative optimality gap $\ll 10^{-4}$. Hence, the proposed market framework has the potential to guarantee grid constraint satisfaction and global optimality for P2P trading enabled decentralized operation of distribution systems. Table VI reports that by enabling P2P trading, the total operating cost of the distribution grid/nodal agents has been reduced by 2.90% in Case 3, and by enabling their reactive power integration, it has been further reduced up to 4.85% in Case 4. Although the total welfare of the P2P participants has been compromised from Case 2 to Case 3 for grid constraint feasibility, it has been improved in Case 4 with the rewards offered for supporting reactive power requirements.

<table>
<thead>
<tr>
<th>Case</th>
<th>Dist./Grid/nodal agents</th>
<th>P2P Producers</th>
<th>P2P Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>879.63</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>2</td>
<td>934.14</td>
<td>−61.67</td>
<td>−66.51</td>
</tr>
<tr>
<td>3</td>
<td>854.12</td>
<td>−54.42</td>
<td>−30.61</td>
</tr>
<tr>
<td>4</td>
<td>836.93</td>
<td>−68.13</td>
<td>−93.51</td>
</tr>
</tbody>
</table>
These results obtained via simulations on a potentially large-scale distribution system, like the modified IEEE 161-bus system, sufficiently illustrate that the proposed market framework (that supports P2P trading enabled decentralized operation of distribution grids) guarantees market equilibrium, grid-constraint feasibility, and global optimality. Further, the performance scales well with the size of the system and the number of market participants.

VI. CONCLUSIONS

In this work, a novel market framework was proposed for P2P energy trading enabled optimal decentralized operation of distribution systems. Therein, nodal and P2P agents were established, and market operations were decentralized while ensuring the agent interactions preserve their autonomy and anonymity of participation, strengthening the privacy concerns. ADMM-based optimization-coordination algorithms were defined for each market participant to execute the market mechanism, which resulted in grid constraint satisfaction, market equilibrium, and global optimality. Case studies were conducted on the modified IEEE 33-bus distribution system and extended for a potentially large-scale distribution system of 161 buses. The results illustrated that P2P energy transactions computed without considering their impact on grid constraints are suboptimal in practice and might lead to infeasible operation as well. The proposed market framework supported grid-constraint-feasible P2P energy trading while achieving global optimality with sufficient accuracy in an acceptable amount of computation time. Further, the market equilibrium of the proposed market framework is also a trade-off between the costs/rewards of ancillary services associated with the respective P2P energy transactions, in addition to marginal costs of producers and marginal welfare of consumers. Moreover, the proposed market framework leverages the benefits of reactive power capabilities and evaluates the costs of the reactive power demand of P2P participants.

In this work, we assume the behaviour of market participants/agents are cooperative, non-strategic, and rational within the market-clearing mechanism, which may not be fully expected in a real-world implementation. We focus on the market-clearing for a single market period of either an hour, 30 minutes, or 15 minutes in the P2P energy market to demonstrate the performance of the proposed method in a more explicit manner. The communication system is considered to be ideal, avoiding communication bandwidth and data transfer speed limitations that can restrict the application for large-scale systems. Addressing these shortcomings is beyond the scope of this work and can be considered in future research to enhance the practicality of the proposed decentralized market framework.

REFERENCES


---

**La hadn a Purage Mohasha Isuru Sampath** (M’21) received the B.Eng. degree of B.Sc. Eng. in Electrical Engineering from University of Moratuwa, Sri Lanka, in 2014. He was working as an electrical engineer in Sri Lanka Ports Authority in 2015. He received the Ph.D. degree from Interdisciplinary Graduate School, Nanyang Technological University (NTU), Singapore, in 2020. Currently, he is a research fellow in Blockchain-based Decentralized Peer-to-Peer Energy Trading project in School of EEE, NTU, Singapore. His research interests include convex optimization, distributed energy trading, modeling and optimization under renewable energy based uncertainties, and optimal power flow.

**Amrit Paudel** (S’16) received the B.E. degree in electrical and electronic engineering from Pokhara University, Nepal in 2012; the M.E. degree in energy engineering from Asian Institute of Technology, Bangkok, Thailand in 2016; and the Ph.D. degree in electrical and electronic engineering from Nanyang Technological University, Singapore in 2020. He is currently a Postdoctoral Research Fellow with the Faculty of Applied Science, School of Engineering, The University of British Columbia, Okanagan, Canada. His current research interest includes electric vehicles and infrastructure planning, microgrid energy management systems, peer-to-peer energy trading, and distribution level electricity market.

**Hung D. Nguyen** (S’12) received the Ph.D. degree in Electric Power Engineering at Massachusetts Institute of Technology (MIT). He is a 2017 Siebel Scholar in energy science. Currently, he is an Assistant Professor in Electrical and Electronic Engineering at NTU, Singapore. His research interests include power system operation and control with Machine Learning, the nonlinearity, dynamics and stability of large scale power systems; DSA/EMS and smart grids.

**Eddy Y. S. Foo** (M’16) received the B.Eng. and Ph.D. degrees in electrical and electronic engineering from Nanyang Technological University, Singapore, in 2009 and 2016, respectively. From 2014 to 2016, he was a Research Engineer with the Cambridge Centre for Advanced Research and Education in Singapore Ltd., an entity under the National Research Foundation’s Campus for Research Excellence and Technological Enterprise Program. Since 2016, he has been a Lecturer with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include multiagent systems, microgrid energy management systems, electricity markets, and renewable energy resources.
Hoay Beng Gooi (LSM, IEEE) received the B.S. degree from National Taiwan University, Taipei, Taiwan in 1978; the M.S. degree from University of New Brunswick, Fredericton, NB in 1980; and the Ph.D. degree from Ohio State University, Columbus, OH in 1983; all in electrical engineering. He worked as Assistant Professor at Lafayette College, Easton, PA during 1983-85 and Senior Engineer at Control Data – Energy Management System Division, Plymouth, MN for about six years before joining Nanyang Technological University (NTU) in 1991, Singapore. He is an Associate Professor with the School of Electrical and Electronic Engineering. He has served as Co-Director of SP Group-NTU Joint Lab since 2020 and Chairman, LMAG, IEEE Singapore since 2021. His current research interests include microgrid energy management systems dealing with energy storage, condition monitoring, electricity market, and spinning reserve.